National
Qualifications
2023

2023 Mathematics

Higher - Paper 1

Finalised Marking Instructions

SQA
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General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6\times6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$ $\quad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\quad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ express second term in differentiable form •² differentiate one term •³ complete differentiation	•¹ $\ldots -10x^{-4}$ stated or implied by •³ •² $\frac{5}{3}x^{\frac{2}{3}}\ldots$ or $\ldots +40x^{-5}$ •³ $\frac{5}{3}x^{\frac{2}{3}}+40x^{-5}$	3

Notes:

1. Where candidates "differentiate over two lines" see Candidates A and B.
2. •³ is only available for differentiating a term with a negative index.
3. Where candidates attempt to integrate throughout, only •¹ is available.

Commonly Observed Responses:

Candidate A - differentiating over two lines	Candidate B - differentiating over two lines
$y=x^{\frac{5}{3}}-\frac{10}{x^4}$ $y=\frac{5}{3}x^{\frac{2}{3}}-10x^{-4}$ •¹ ✓ $y=\frac{5}{3}x^{\frac{2}{3}}+40x^{-5}$ •² ✓ •³ ✗	$y=x^{\frac{5}{3}}-\frac{10}{x^4}$ $y=\frac{5}{3}x^{\frac{2}{3}}-10x^{-4}$ •¹ ✓ $y=\frac{5}{3}x^{\frac{2}{3}}+40x^{-3}$ •² ✓ •³ ✗
Candidate C ⋮ $\frac{5}{3}x^{\frac{2}{3}}+40x^{-5}+c$ •³ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ find midpoint of PQ	•¹ $(4,3)$	**4**
			•² calculate gradient of PQ	•² $-\frac{1}{2}$ or $-\frac{6}{12}$	
			•³ state perpendicular gradient	•³ 2 stated or implied by •⁴	
			•⁴ determine equation of perpendicular bisector	•⁴ $y = 2x - 5$	

Notes:

1. •⁴ is only available as a consequence of using a perpendicular gradient **and** a midpoint.
2. The gradient of the perpendicular bisector must appear in fully simplified form at •³ or •⁴ stage for •⁴ to be awarded.
3. At •⁴, accept $2x - y = 5$, $y - 2x = -5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			**Method 1** $\bullet^1$ apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$ $\bullet^2$ write in exponential form $\bullet^3$ process for x	**Method 1** $\bullet^1$ $\log_5 \frac{x}{3} \ldots$ $\bullet^2$ $\frac{x}{3} = 5^2$ $\bullet^3$ 75	3
			Method 2 $\bullet^1$ apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$ $\bullet^2$ apply $m \log_5 x = \log_5 x^m$ $\bullet^3$ process for x	**Method 2** $\bullet^1$ $\log_5 \frac{x}{3} \ldots$ $\bullet^2$ $\ldots = \log_5 5^2$ $\bullet^3$ 75	3

Notes:

1. Each line of working must be equivalent to the line above within a valid strategy, however see Candidates A and B for exceptions.
2. Where candidates do not use exponentials at $\bullet^2$, $\bullet^3$ is not available - see Candidate C.

Commonly Observed Responses:

Candidate A - incorrect exponential		Candidate B	
$\log_5 \frac{x}{3} = 2$	$\bullet^1$ ✓	$\log_5 3x = 2$	$\bullet^1$ ✗
$\frac{x}{3} = 2^5$	$\bullet^2$ ✗	$3x = 5^2$	$\bullet^2$ $\boxed{\checkmark_1}$
$x = 96$	$\bullet^3$ $\boxed{\checkmark_1}$	$x = \frac{25}{3}$	$\bullet^3$ $\boxed{\checkmark_1}$

Candidate C - no use of exponentials			
$\log_5 \frac{x}{3} = 2$	$\bullet^1$ ✓		
$\frac{x}{3} = 10$	$\bullet^2$ ✗		
$x = 30$	$\bullet^3$ ✗		

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] find $\cos p$	•[1] $\frac{3}{5}$	1
			•[2] find $\cos q$	•[2] $\frac{3}{\sqrt{45}}\left(=\frac{1}{\sqrt{5}}\right)$	1

Notes:

1. Accept $\frac{3}{3\sqrt{5}}$ for •[2].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] select appropriate formula and express in terms of p and q •[4] substitute into addition formula •[5] evaluate $\cos(p+q)$	•[3] $\cos p\cos q-\sin p\sin q$ •[4] $\frac{3}{5}\times\frac{3}{\sqrt{45}}-\frac{4}{5}\times\frac{6}{\sqrt{45}}$ •[5] $-\frac{3}{\sqrt{45}}\left(=-\frac{1}{\sqrt{5}}\right)$	3

Notes:

2. Award •[3] for candidates who write $\cos\left(\frac{3}{5}\right)\times\cos\left(\frac{3}{\sqrt{45}}\right)-\sin\left(\frac{4}{5}\right)\times\sin\left(\frac{6}{\sqrt{45}}\right)$. •[4] and •[5] are unavailable.
3. For any attempt to use $\cos(p+q)=\cos p\pm\cos q$, •[4] and •[5] are unavailable.
4. •[5] is only available if either the surd part or the non-surd part of the fraction is simplified as far as possible. Accept $-\frac{3}{\sqrt{45}}$, $-\frac{\sqrt{45}}{15}$, $-\frac{15}{15\sqrt{5}}$ or answers obtained on follow through which do not require simplification. Do not accept $-\frac{15}{5\sqrt{45}}$.
5. •[5] is only available for an answer expressed as a single fraction.

Commonly Observed Responses:

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>5.</td><td></td><td></td><td>•1 use the discriminant

•2 apply condition and express in standard quadratic form

•3 process for p</td><td>•1 $(3p-2)^2-4\times2\times p$

•2 $9p^2-20p+4=0$

•3 $\frac{2}{9},2$</td><td>3</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. Where candidates states an incorrect condition, •2 is not available. However, •3 is available for finding the roots of the quadratic - see Candidate B.
2. Where x appears in any expression, no further marks are available.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="4">Candidate A
(For equal roots) $b^2-4ac=0$
$(3p-2)^2-4\times2\times p$ •1 ✓
$9p^2-20p+4$ •2 ✓
$p=\frac{2}{9},\ 2$ •3 ✓</td><td colspan="2">Candidate B
(For equal roots) $b^2-4ac>0$ •2 ✗
$(3p-2)^2-4\times2\times p$ •1 ✓
$9p^2-20p+4=0$
$p=\frac{2}{9},\ 2$ •3 $\boxed{✓_1}$
⋮</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
6.			$\bullet^1$ express second term in integrable form	$\bullet^1$ $\ldots - 6x^{\frac{1}{2}}$	4
			$\bullet^2$ integrate one term	$\bullet^2$ $\frac{2}{6}x^6 \ldots$ or $\ldots - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$	
			$\bullet^3$ integrate other term	$\bullet^3$ $\ldots - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ or $\frac{2}{6}x^6 \ldots$	
			$\bullet^4$ complete integration	$\bullet^4$ $\frac{1}{3}x^6 - 4x^{\frac{3}{2}} + c$	

Notes:

1. The mark for integrating the final term is only available if candidates integrate a term with a fractional index.
2. Do not penalise the appearance of an integral sign and/or dx throughout.
3. Do not penalise the omission of '$+c$' at $\bullet^2$ or $\bullet^3$.
4. All coefficients must be simplified at $\bullet^4$ stage for $\bullet^4$ to be awarded.
5. Accept $\frac{x^6 - 12x^{\frac{3}{2}}}{3} + c$ for $\bullet^4$ but do not accept $\frac{2x^6 - 24x^{\frac{3}{2}}}{6} + c$.
6. $\bullet^2$, $\bullet^3$ and $\bullet^4$ are not available within an invalid strategy.

Commonly Observed Responses:

Candidate A

$\int \left(2x^5 - 6x^{\frac{1}{2}}\right) dx$ $\bullet^1$ ✓

$= \frac{2x^6}{6} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + c$ $\bullet^2$ ✓ $\bullet^3$ ✓

$= \boxed{\frac{2x^6}{6}} - 4x^{\frac{3}{2}} + c$

$= \frac{1}{3}x^6 - \boxed{4\sqrt{x}} + c$ $\bullet^4$ ✗

$\bullet^4$ cannot be awarded over two lines of working

Candidate B - integrating over two lines

$\frac{2x^6}{6} - 6x^{\frac{1}{2}}$ $\bullet^1$ ✓

$= \frac{2x^6}{6} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + c$ $\bullet^2$ ✓ $\bullet^3$ ✗

$= \frac{1}{3}x^6 - 4x^{\frac{3}{2}} + c$ $\bullet^4$ $\boxed{✓_1}$

Candidate C - insufficient evidence

$\int 2x^5 - 6x^{\frac{1}{2}} dx$ $\bullet^1$ ✓

$\frac{1}{3}x^6 - 9x^{\frac{3}{2}} + c$ $\bullet^2$ ✓ $\bullet^3$ ^ $\bullet^4$ ✗

Candidate D

$\vdots$ $\bullet^1$ ✓

$= \frac{1}{3}x^6 - 4x^{\frac{3}{2}}$ $\bullet^2$ ✓ $\bullet^3$ ✓

$= \frac{1}{3}x^6 - 4\sqrt{x^3} + c$ $\bullet^4$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ use laws of logs •² evaluate log	•¹ $\log_2 \frac{5}{40}$ •² -3	**2**

Notes:

1. Do not penalise the omission of the base of the logarithm at •¹.
2. Correct answer with no working, award 0/2.

Commonly Observed Responses:

Candidate A - introducing a variable

$\log_2\left(5\times\frac{1}{40}\right)$ •¹ ✓

$\log_2 \frac{1}{8}$

$2^x = \frac{1}{8}$

$x = -3$ •² ✓

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ state range	•³ $0 < a < 1$	**1**

Notes:

3. At •³ accept "$a > 0$ and $a < 1$" or "$a > 0$, $a < 1$ ".

Commonly Observed Responses:

<table>
<tr><th>Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>8.</td><td>$\bullet^1$ start to differentiate</td><td>$\bullet^1$ $3x^2\ldots$ or $\ldots+6x\ldots$ or $\ldots-9$</td><td>6</td></tr>
<tr><td></td><td>$\bullet^2$ complete differentiation and equate to 0</td><td>$\bullet^2$ $3x^2+6x-9=0$</td><td></td></tr>
<tr><td></td><td>$\bullet^3$ solve for x</td><td>$\bullet^3$ $\bullet^4$
$\bullet^3$ -3 and 1</td><td></td></tr>
<tr><td></td><td>$\bullet^4$ process for y</td><td>$\bullet^4$ 32 and 0</td><td></td></tr>
<tr><td></td><td>$\bullet^5$ construct nature table(s)</td><td>$\bullet^5$ $\bullet^5$ $\bullet^6$
<table>
<tr><td>x</td><td>$\ldots$</td><td>-3</td><td>$\ldots$</td><td>1</td><td>$\ldots$</td></tr>
<tr><td>$f'(x)$</td><td>+</td><td>0</td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>shape</td><td>/</td><td>—</td><td>\</td><td>—</td><td>/</td></tr>
</table></td><td></td></tr>
<tr><td></td><td>$\bullet^6$ interpret and state conclusions</td><td>$\bullet^6$ max at $(-3, 32)$; min at $(1, 0)$</td><td></td></tr>
</table>

Notes:

1. For a numerical approach award 0/6.
2. $\bullet^2$ is only available if '$=0$' appears at the $\bullet^2$ stage or in working leading to $\bullet^3$, however see Candidates A and B.
3. Candidates who equate their derivative to 0, may use division by 3 as a strategy - see candidates B, C and D.
4. $\bullet^3$ is available to candidates who factorise **their** derivative from $\bullet^2$ as long as it is of equivalent difficulty.
5. $\bullet^3$ and $\bullet^4$ may be awarded vertically.
6. $\bullet^5$ is not available where any errors are made in calculating values of $f'(x)$.
7. $\bullet^5$ and $\bullet^6$ may be awarded vertically.
8. $\bullet^6$ is still available in cases where a candidates table of signs does not lead legitimately to a maximum/minimum shape.
9. Candidates may use the second derivative - see Candidates E and F.
10. Accept "max when $x=-3$" **and** "min when $x=1$" for $\bullet^6$.

Commonly Observed Responses:

Candidate A Stationary points when $f'(x)=0$ $f'(x)=3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ✓ $f'(x)=3(x+3)(x-1)$ $x=-3,\ 1$ $\bullet^3$ ✓	**Candidate B** Stationary points when $f'(x)=0$ $f'(x)=3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ✓ ⋮ $f'(x)=(x+3)(x-1)$ $x=-3,\ 1$ $\bullet^3$ ✓
Candidate C - division by 3 $3x^2+6x-9=0$ $\bullet^1$ ✓ $\bullet^2$ ✓ $x^2+2x-3=0$ $x=-3,\ 1$ $\bullet^3$ ✓	**Candidate D - derivative never equated to 0** $3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ^ $x^2+2x-3=0$ $x=-3,\ 1$ $\bullet^3$ $\boxed{\checkmark_1}$

Question	Generic scheme	Illustrative scheme	Max mark
8.(continued)			

Commonly Observed Responses:

Candidate E - second derivative

$f''(x) = 6x + 6$

•5 ✓	•6 ✓	
$f''(-3) < 0$	$f''(1) > 0$	•5 ✓
so max at $(-3,32)$	so min at $(1,0)$	•6 ✓

Candidate F - second derivative

$f''(x) = 6x + 6$

•5 ✓	•6 ✓	
$f''(-3) = -12$,	$f''(1) = 12$	•5 ✓
$-12 < 0$	$12 > 0$	
so max at $(-3,32)$	so min at $(1,0)$	•6 ✓

For the table of signs for a derivative, accept:

x	-3^-	-3	-3^+
$f'(x)$	$+$	0	$-$
Slope or shape	/	—	\

AND

x	1^-	1	1^+
$f'(x)$	$-$	0	$+$
Slope or shape	\	—	/

x	$\rightarrow$	-3	$\rightarrow$
$f'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Arrows are taken to mean 'in the neighbourhood of'

AND

x	$\rightarrow$	1	$\rightarrow$
$f'(x)$	$-$	0	$+$
Slope or shape	\	—	/

Arrows are taken to mean 'in the neighbourhood of'

x	a	-3	b
$f'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Where $a < -3$ and $-3 < b < 1$

AND

x	c	1	d
$f'(x)$	$-$	0	$+$
Slope or shape	\	—	/

Where $-3 < c < 1$ and $d > 1$

For the table of signs for a derivative, accept:

x	$\rightarrow$	-3	$\rightarrow$	1	$\rightarrow$
$f'(x)$	$+$	0	$-$	0	$+$
Slope or shape	/	—	\	—	/

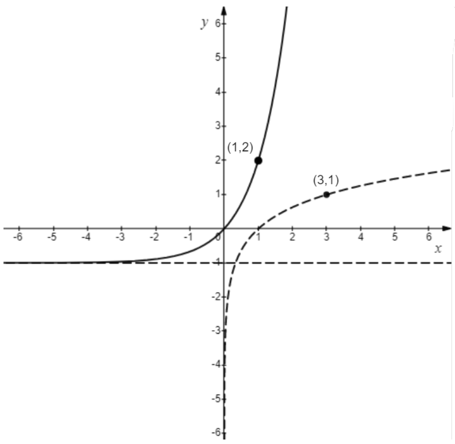
Since the function is continuous $-3 \rightarrow 1$ is acceptable

x	a	-3	b	1	c
$f'(x)$	$+$	0	$-$	0	$+$
Slope or shape	/	—	\	—	/

Since the function is continuous $-3 < b < 1$ is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $f'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $f'(x)$ are: f', $\frac{df}{dx}$, $\frac{dy}{dx}$, $3x^2 + 6x - 9$ and $3(x+3)(x-1)$

  but NOT $x^2 + 2x - 3$ or $(x+3)(x-1)$.

Question			Generic scheme	Illustrative scheme	Max mark
9.			•[1] graph reflected in $y = x$ •[2] vertical translation of "-1" unit following a reflection in $y = x$ identifiable from graph •[3] sketch of required function	•[1] a concave up curve above the x-axis for $x > 0$ •[2] curve passing through (0,0) and (1,2) •[3] curve approaches the line $y = -1$ from above as $x \to -\infty$ 	**3**

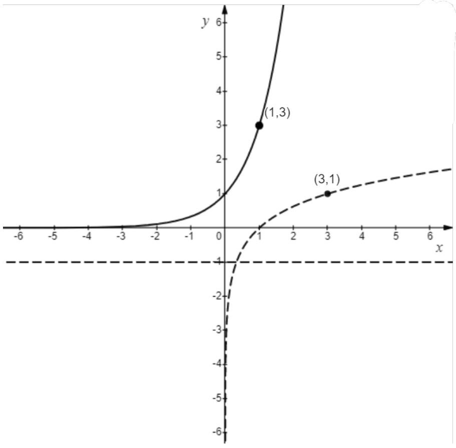
Notes:

1. For •[1] accept any graph of a function which is concave up within the first quadrant.
2. •[1] is only available where the candidate has attempted to reflect the given curve in the line $y = x$.
3. •[3] is only available where the curve passes through (0,0) and (1,2).
4. The line $y = -1$ does not need to be shown.
5. For a rotation, award 0/3 – for example see Candidate D.

Question	Generic scheme	Illustrative scheme	Max mark
9.(continued)			

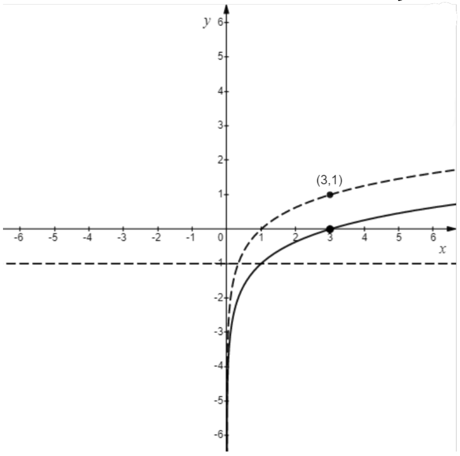
Commonly Observed Responses:

Candidate A - reflection only



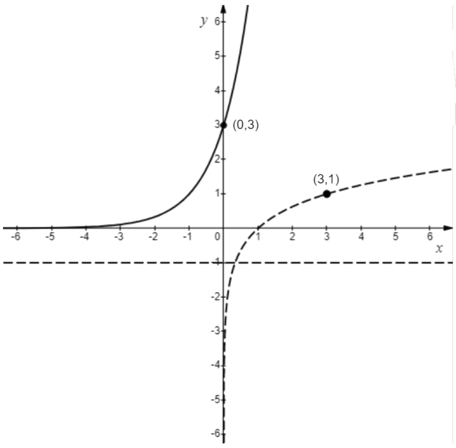
•1 ✓ •2 × •3 ×

Candidate B - translation only



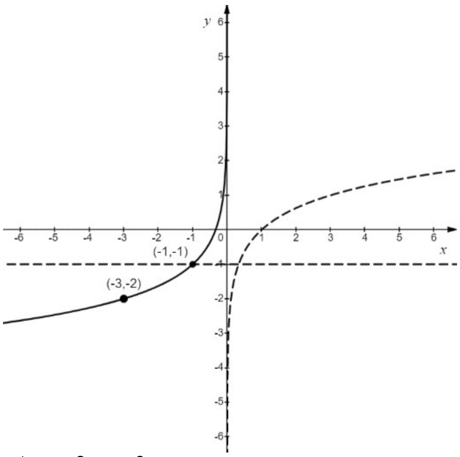
•1 × •2 × •3 ×

Candidate C - incorrect order of transformations



•2 × •1 $\boxed{✓_1}$ •3 ×

Candidate D - rotation



•1 × •2 × •3 ×

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•¹ use -5 in synthetic division or evaluation of quartic	•¹ $\begin{array}{r\|rrrrr} -5 & 1 & 3 & -7 & 9 & -30 \\ \hline & 1 & & & & \boxed{} \end{array}$ or $(-5)^4+3\times(-5)^3-7\times(-5)^2$ $+9\times(-5)-30$	2
			•² complete division/evaluation and interpret result	•² $\begin{array}{r\|rrrrr} -5 & 1 & 3 & -7 & 9 & -30 \\ & & -5 & 10 & -15 & 30 \\ \hline & 1 & -2 & 3 & -6 & \boxed{0} \end{array}$ $\text{Remainder}=0\ \therefore (x+5)$ is a factor **OR** $f(-5)=0\ \therefore (x+5)$ is a factor	

Notes:

1. Communication at •² must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •²:
   - '$f(-5)=0$ so $(x+5)$ is a factor'
   - 'since remainder $=0$, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for •²:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=-5$ is a factor', '... is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Candidate A - grid method

	x^3			
x	x^4	$-2x^3$		
5	$5x^3$			

•¹ ✓

	x^3	$-2x^2$	$+3x$	-6
x	x^4	$-2x^3$	$+3x^2$	$-6x$
5	$5x^3$	$-10x^2$	$+15x$	-30

with no remainder

$\therefore (x+5)$ is a factor •² ✓

Candidate B - grid method

	x^3			
x	x^4	$-2x^3$		
5	$5x^3$			

•¹ ✓

	x^3	$-2x^2$	$+3x$	-6
x	x^4	$-2x^3$	$+3x^2$	$-6x$
5	$5x^3$	$-10x^2$	$+15x$	-30

$\therefore (x+5)(x^3-2x^2+3x-6)=x^4+3x^3-7x^2+9x-30$

$\therefore (x+5)$ is a factor •² ✓

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>10.</td><td>(b)</td><td></td><td>•3 identify cubic and attempt to factorise</td><td>•3 eg
$\begin{array}{r|rrrr} 1 & 1 & -2 & 3 & -6 \\ & & 1 & -1 & \\ \hline & 1 & -1 & & \boxed{} \end{array}$
or
$\begin{array}{r|rrrr} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & \\ \hline & 1 & 0 & & \boxed{} \end{array}$</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td>•4 find second factor</td><td>•4
$\begin{array}{r|rrrr} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & 6 \\ \hline & 1 & 0 & 3 & \boxed{0} \end{array}$
leading to $(x-2)$ or $x=2$</td><td></td></tr>
<tr><td></td><td></td><td></td><td>•5 identify quadratic</td><td>•5 x^2+3</td><td></td></tr>
<tr><td></td><td></td><td></td><td>•6 interpret lack of solutions of quadratic</td><td>•6 $b^2-4ac=-12<0$
$\therefore$ no (further real) solutions
OR
$x^2=-3$ or $\cancel{x^2=3}$
$\therefore$ no (further real) solutions</td><td></td></tr>
<tr><td></td><td></td><td></td><td>•7 state solutions</td><td>•7 $x=-5,\ x=2$</td><td></td></tr>
</table>

Notes:

4. Candidates who arrive at $(x+5)(x-2)(x^2+3)$ by using algebraic long division or by inspection gain •3, •4 and •5.
5. Evidence for •6 may appear in the quadratic formula.
6. At •6 accept interpretations such as "no further roots", "no solutions" and "cannot factorise further" with justification.
7. At •6 accept $x=\sqrt{-3}$ **leading to** "not possible" and "not real".
8. Where there is no reference to b^2-4ac accept '$-12<0$ so no real roots' **with the remaining roots stated** for •6 - see candidates E and F.
9. Do not accept any of the following for •6:
    - $(x+5)(x-2)(x^2+3)$ no further roots/cannot factorise further.
    - $(x+5)(x-2)(\dots\ \dots)(\dots\ \dots)$ no further roots/cannot factorise further.
10. Where the quadratic factor obtained at •5 can be factorised, •6 and •7 are not available.
11. •7 is only available where •6 has been awarded.

Question	Generic scheme	Illustrative scheme	Max mark
10.(continued)			

Commonly Observed Responses:

Candidate C

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$b^2-4ac=0-12<0$ $\bullet^6$ ✓

so no solutions

$x=-5,\ x=2$ $\bullet^7$ ✓

Candidate D

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$b^2-4ac<0$ $\bullet^6$ ^

so no solutions

$x=-5,\ x=2$ $\bullet^7$ $\boxed{✓2}$

Candidate E

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$-12<0$ $\bullet^6$ ✓

so no solutions

$x=-5,\ x=2$ $\bullet^7$ ✓

Candidate F

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$-12<0$

so no solutions $\bullet^6$ ^ $\bullet^7$ ^

Candidate G - grid method

(a)

	x^3	$-2x^2$	$3x$	-6
x	x^4	$-2x^3$	$+3x^2$	$-6x$
5	$5x^3$	$-10x^2$	$+15x$	-30

(b)

	x^2	…	…
x	x^3	…	…
…	…	…	…

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	x^2	$0x$	3
x	x^3	0	$3x$
-2	$-2x^2$	0	-6

$\bullet^4$ ✓

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$b^2-4ac=-12<0$

$\therefore$ no more solutions $\bullet^6$ ✓

$x=-5,\ x=2$ $\bullet^7$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•[1] integrate	•[1] $-5\cos x - 3\sin x$	**3**
			•[2] substitute limits	•[2] $\left[-5\cos\pi - 3\sin\pi\right] - \left[-5\cos\frac{\pi}{2} - 3\sin\frac{\pi}{2}\right]$	
			•[3] evaluate integral	•[3] 8	

Notes:

1. Where candidates make no attempt to integrate or use another invalid approach award 0/3 - see Candidate A. However see also Candidates B to F.
2. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign.
3. Candidates who change the limits to degrees before integrating cannot gain •[1]. However, •[2] and •[3] are still available.
4. •[3] is only available where candidates have considered both limits within a trigonometric function.
5. The minimum acceptable response for •[2] is $5-(-3)$.

Commonly Observed Responses:

Candidate A - introducing a power

Eg $5\sin x^2 - 3\cos x^2$ •[1] ✗ •[2] ✗ •[3] ✗

⋮

Candidate B - differentiating in full

$5\cos x + 3\sin x$ •[1] ✗

$(5\cos\pi + 3\sin\pi) - \left(5\cos\frac{\pi}{2} + 3\sin\frac{\pi}{2}\right)$ •[2] $\boxed{\checkmark_2}$

-8 •[3] $\boxed{\checkmark_1}$

Candidate C - integrating one term

$5\cos x - 3\sin x$ •[1] ✗

$(5\cos\pi - 3\sin\pi) - \left(5\cos\frac{\pi}{2} - 3\sin\frac{\pi}{2}\right)$ •[2] $\boxed{\checkmark_1}$

-2 •[3] $\boxed{\checkmark_1}$

Candidate D - integrating one term

$-5\cos x + 3\sin x$ •[1] ✗

$(-5\cos\pi + 3\sin\pi) - \left(-5\cos\frac{\pi}{2} + 3\sin\frac{\pi}{2}\right)$ •[2] $\boxed{\checkmark_1}$

2 •[3] $\boxed{\checkmark_1}$

Candidate E - integrating one term

Eg $5\sin x - 3\sin x$ •[1] ✗

$(5\sin\pi - 3\sin\pi) - \left(5\sin\frac{\pi}{2} - 3\sin\frac{\pi}{2}\right)$ •[2] $\boxed{\checkmark_1}$

-2 •[3] $\boxed{\checkmark_2}$

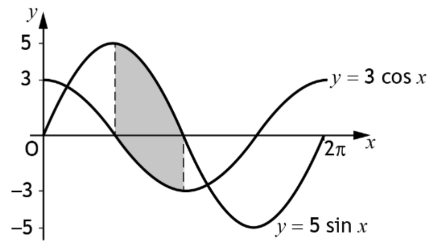
Mark 3 is not of equivalent difficulty - only 2 exact values

Candidate F - obtaining other expressions of the form $a\sin x + b\cos x$

Eg $-\frac{1}{5}\cos x - \frac{1}{3}\sin x$ •[1] ✗

$\left(-\frac{1}{5}\cos\pi - \frac{1}{3}\sin\pi\right) - \left(-\frac{1}{5}\cos\frac{\pi}{2} - \frac{1}{3}\sin\frac{\pi}{2}\right)$ •[2] $\boxed{\checkmark_2}$

$\frac{8}{15}$ •[3] $\boxed{\checkmark_1}$

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		•⁴ identify boundaries and shade area	•⁴ 	**1**

Notes:

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
12.	**Method 1**	**Method 1**	3
	$\bullet^1$ identify common factor	$\bullet^1$ $-2\left(x^2+6x\ldots\right.$ stated or implied by $\bullet^2$	
	$\bullet^2$ complete the square	$\bullet^2$ $-2(x+3)^2\ldots$	
	$\bullet^3$ process for c and write in required form	$\bullet^3$ $-2(x+3)^2+25$	
	Method 2	**Method 2**	
	$\bullet^1$ expand completed square form	$\bullet^1$ $ax^2+2abx+ab^2+c$ stated or implied by $\bullet^2$	
	$\bullet^2$ equate coefficients	$\bullet^2$ $a=-2$, $2ab=-12$, and $ab^2+c=7$	
	$\bullet^3$ process for b and c and write in required form	$\bullet^3$ $-2(x+3)^2+25$	

Notes:

1. $-2(x+3)^2+25$ with no working gains $\bullet^1$ and $\bullet^2$ only. However, see Candidate E.
2. $\bullet^1$ and $\bullet^3$ are not available in cases where $a>0$. For example, see Candidate F.

Commonly Observed Responses:

Candidate A

$-2\left(x^2+6\right)+7$

$-2\left((x+3)^2-9\right)+7$ $\bullet^1$ ✓ $\bullet^2$ ✓

$-2(x+3)^2+25$ $\bullet^3$ ✓

See the exception to marking principle (h)

Candidate B

$ax^2+2abx+ab^2+c$ $\bullet^1$ ✓

$a=-2$, $2ab=-12$, $ab^2+c=7$ $\bullet^2$ ✓

$b=3$, $c=25$ $\bullet^3$ ^

$\bullet^3$ is lost as answer is not in completed square form

Candidate C

$-2\left(x^2+12x\right)+7$ $\bullet^1$ ✗

$-2\left((x+6)^2-36\right)+7$ $\bullet^2$ $\boxed{\checkmark_1}$

$-2(x+6)^2+79$ $\bullet^3$ $\boxed{\checkmark_1}$

Candidate D

$-2\left((x+6)^2-36\right)+7$ $\bullet^1$ ✗ $\bullet^2$ ✗

$-2(x+6)^2+79$ $\bullet^3$ $\boxed{\checkmark_1}$

Candidate E

$-2(x+3)^2+25$ $\bullet^1$ ✓ $\bullet^2$ ✓

Check: $=-2\left(x^2+6x+9\right)+25$

$=-2x^2-12x-18+25$

$=-2x^2-12x+7$ $\bullet^3$ ✓

Candidate F

$-2x^2-12x+7$

$=2x^2+12x-7$ $\bullet^1$ ✗

$=2\left(x^2+6x\ldots\right.$

$=2(x+3)^2\ldots$ $\bullet^2$ $\boxed{\checkmark_1}$

$=-2(x+3)^2\ldots$ $\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)	(i)	•¹ state exact value	•¹ $\sqrt{3}$	**1**
		(ii)	•² interpret notation •³ state expression for $f(g(x))$	•² $f(2x)$ or $2\sin(g(x))$ •³ $2\sin 2x$	**2**

Notes:

1. For $f(g(x)) = 2\sin 2x$ without working, award both •² and •³.
2. Working for (a)(ii) may be found in (a)(i)

Commonly Observed Responses:

Candidate A (a)(ii) $f(g(x)) = 4\sin x$ •² ✗ •³ $\boxed{✓_1}$	**Candidate B - Beware of "2 attempts"** $f(g(x)) = 2\sin x$ •² ✗ •³ ✗ $f(2x) = 2\sin 2x$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•⁴ find the value of $\sin p$	•⁴ $\frac{1}{6}$	**1**
		(ii)	•⁵ expand $f(g(p))$ using double angle formula •⁶ find value of $\cos p$ •⁷ substitute and determine exact value	•⁵ $2\times 2\sin p\cos p$ or $4\sin p\cos p$ **stated explicitly** •⁶ $\frac{\sqrt{35}}{6}$ •⁷ $2\times 2\times\frac{1}{6}\times\frac{\sqrt{35}}{6}$ leading to $\frac{\sqrt{35}}{9}$	**3**

Notes:

1. •⁵ is not available for expansions which do not involve p. •⁶ and •⁷ are still available.
   However, accept $\sin^{-1}\left(\frac{1}{6}\right)$ in place of p - see Candidate C.
2. •⁷ is only available as a consequence of substituting into a valid formula from •⁵.
3. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Candidate C

$f(g(p)) = 4\sin\left(\sin^{-1}\left(\frac{1}{6}\right)\right)\cos\left(\sin^{-1}\left(\frac{1}{6}\right)\right)$ •⁵ ✓

$4\times\frac{1}{6}\times\frac{\sqrt{35}}{6}$ •⁶ ✓

$\frac{\sqrt{35}}{9}$ •⁷ ✓

[END OF MARKING INSTRUCTIONS]

H

National
Qualifications
2023

2023 Mathematics

Higher - Paper 2

Finalised Marking Instructions

SQA
©

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] find gradient of QR •[2] use property of perpendicular lines •[3] determine equation of altitude	•[1] $-\frac{1}{3}$ or $-\frac{5}{15}$ •[2] 3 •[3] $y = 3x - 16$	**3**

Notes:

1. •[3] is only available to candidates who find and use a perpendicular gradient.
2. The gradient of the perpendicular bisector must appear in fully simplified form at •[2] or •[3] stage for •[3] to be awarded - see Candidate B.
3. •[3] is not available as a consequence of using the midpoint of QR and the point P.
4. At •[3], accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A - BEWARE	Candidate B - unsimplified gradient
Correct equation from incorrect substitution $m = \frac{13-(-2)}{3-(8)} = 3$ •[1] ✗ •[2] ^ $y = 3x - 16$ •[3] ✗	$m = -\frac{5}{15}$ •[1] ✓ $m_{\perp} = \frac{15}{5}$ •[2] ✓ $15x - 5y - 80 = 0$ •[3] ^

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] determine gradient of the line •[5] use $m = \tan\theta$ to find the angle	•[4] $m = \frac{1}{2}$ or $\tan\theta = \frac{1}{2}$ •[5] $26\cdot6°$ or 0.4636... radians	**2**

Notes:

5. Do not penalise the omission of units at •[5].
6. Accept any answers which round to 27° or 0.46 radians.
7. For 27° or 0.46 radians without working award 2/2.
8. Where candidates find the angle of the altitude or other sides with the positive direction of the x-axis only •[5] is available.

Commonly Observed Responses:

Candidate C - no reference to tan	Candidate D - BEWARE
$m = \frac{4}{8}$ •[4] ✓ $26.6°$ •[5] ✓	$m = \frac{1}{2}$ •[4] ✓ $\theta = \tan\frac{1}{2}$ •[5] ✗ $\theta = 26.6°$ Stating $\tan$ rather than $\tan^{-1}$ See general marking principle (g)
Candidate E $\tan^{-1}(3) = 72°$ •[4] ✗ •[5] $\boxed{\checkmark_1}$	

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] calculate y-coordinate •[2] differentiate •[3] calculate the gradient •[4] find equation of line	•[1] -1 •[2] $10x^4-3$ •[3] 7 •[4] $y=7x-8$	4

Notes:

1. Only •[1] is available to candidates who integrate.
2. •[4] is only available where candidates attempt to find the gradient by substituting into their derivative.
3. The appearance of $10x^4-3$ gains •[2].
4. •[3] is not available for $y=7$. However, where 7 is then used as the gradient of the straight line, •[3] may be awarded - see Candidates B, C & D.
5. •[4] is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A		**Candidate B - incorrect notation**	
$\frac{dy}{dx}=10x^4-3$	•[2] ✓	$y=-1$	•[1] ✓ - BoD
$y=7$	•[1] ✗	$y=10x^4-3$	•[2] ✓
$m=-3$	•[3] ✗	$y=7$	•[3] ✓ - BoD
$y=-3x+10$	•[4] $\boxed{✓_2}$	$y+1=7(x-1)$	
		$y=7x-8$	•[4] ✓
Candidate C - use of values in equation		**Candidate D - incorrect notation**	
$y=-1$	•[1] ✓ - BoD	$y=-1$	•[1] ✓ - BoD
$\frac{dy}{dx}=10x^4-3$	•[2] ✓	$\frac{dy}{dx}=10x^4-3$	•[2] ✓
$\frac{dy}{dx}=7$	•[3] ✓	$y=7$	•[3] ✗
$y=7$		**Evidence for •[3] would need to appear in the equation of the line**	
$y+1=7(x-1)$			
$y=7x-8$	•[4] ✓		
Candidate E			
$y=-1$	•[1] ✓		
$\frac{dy}{dx}=10x^4-3=0$	•[2] ✓		
$10(1)^4-3=0$	•[3] ✗		
$m=7$			
$y=7x-8$	•[4] $\boxed{✓_1}$		

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] start to integrate •[2] complete integration	•[1] $7\sin\left(4x+\frac{\pi}{3}\right)\ldots$ •[2] $\ldots\times\frac{1}{4}+c$	**2**

Notes:

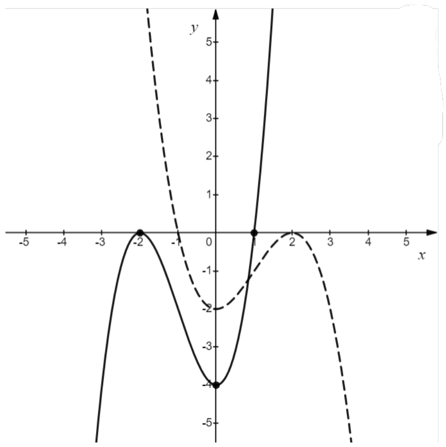
1. Award •[1] for any appearance of $(+)7\sin\left(4x+\frac{\pi}{3}\right)$ regardless of any constant multiplier.
2. Candidates who work in degrees from the start cannot gain •[1], however •[2] is still available - see Candidate C.
3. Where candidates use any other invalid approach, eg $7\sin\left(4x+\frac{\pi}{3}\right)^2$, $\int\left(7\cos 4x+\cos\frac{\pi}{3}\right)dx$ or $7\sin 4x+\frac{\pi}{3}$ award 0/2. However, see Candidate E.
4. Do not penalise the appearance of an integral sign and/or dx throughout.

Commonly Observed Responses:

Candidate A - using addition formula $\int\left(7\cos 4x\cos\frac{\pi}{3}-7\sin 4x\sin\frac{\pi}{3}\right)dx$ $=\frac{7}{4}\sin 4x\cos\frac{\pi}{3}+\frac{7}{4}\cos 4x\sin\frac{\pi}{3}\ldots$ •[1] ✓ $=\frac{7}{4}\sin 4x\left(\frac{1}{2}\right)+\frac{7}{4}\cos 4x\left(\frac{\sqrt{3}}{2}\right)+c$ •[2] ✓	**Candidate B** $\frac{7}{4}\sin\left(4x+\frac{\pi}{3}\right)$ •[1] ✓ $=\frac{7}{4}\sin\left(4x+\frac{\pi}{3}\right)+c$ •[2] ✓
Candidate C - working in degrees $\int 7\cos(4x+60)\,dx$ $=7\sin(4x+60)\times\frac{1}{4}+c$ •[1] ✗ •[2] $\boxed{✓_1}$	**Candidate D - integrating over two lines** $7\sin\left(4x+\frac{\pi}{3}\right)$ •[1] ✓ $=\frac{7}{4}\sin\left(4x+\frac{\pi}{3}\right)+c$ •[2] ✗
Candidate E - integrating in part $-\frac{7}{4}\sin\left(4x+\frac{\pi}{3}\right)+c$ •[1] ✗ •[2] $\boxed{✓_1}$	**Candidate F - insufficient evidence of integration** $\frac{7}{4}\cos\left(4x+\frac{\pi}{3}\right)+c$ •[1] ✗ •[2] ✗

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>4.</td><td></td><td></td><td>•[1] reflect in the y-axis

•[2] apply appropriate vertical scaling</td><td>•[1] cubic graph with max at $(-2, 0)$ and passing through $(1, 0)$

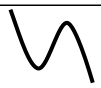
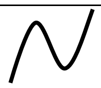
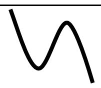
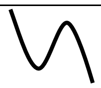
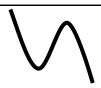
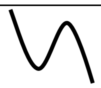
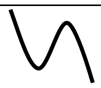
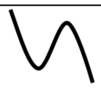
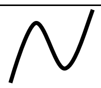
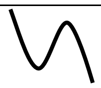
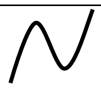
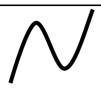
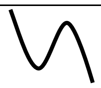
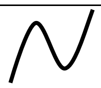
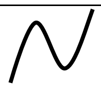
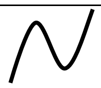
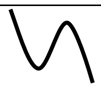
•[2]
</td><td>2</td></tr>
</table>

Notes:

1. Where candidates do not sketch a cubic function award 0/2.
2. For transformations of the form $f(-x)+k$ or $-f(x+k)$ award 0/2.
3. If the transformation has not been applied to all coordinates, award 0/2.

Question	Generic scheme	Illustrative scheme	Max mark
4.	**(continued)**		

Commonly Observed Responses:

Function	Transformation of $(-1,0)$ and $(2,0)$	Transformation of $(0,-2)$	Shape	Award
Incorrect orientation	$(-2,0)$ and $(1,0)$	$(0,-4)$		0/2
$-2f(x)$	$(-1,0)$ and $(2,0)$	$(0,4)$		1/2
$-2f(-x)$	$(-2,0)$ and $(1,0)$	$(0,4)$		1/2
$-2f(-2x)$	$(-1,0)$ and $(\frac{1}{2},0)$	$(0,4)$		0/2
$-2f\left(-\frac{x}{2}\right)$	$(-4,0)$ and $(2,0)$	$(0,4)$		0/2
$2f(x)$	$(-1,0)$ and $(2,0)$	$(0,-4)$		1/2
$2f(2x)$	$(-\frac{1}{2},0)$ and $(1,0)$	$(0,-4)$		1/2
$2f\left(\frac{x}{2}\right)$	$(-2,0)$ and $(4,0)$	$(0,-4)$		1/2
$2f\left(-\frac{x}{2}\right)$	$(-4,0)$ and $(2,0)$	$(0,-4)$		1/2
$2f(x-1)$	$(0,0)$ and $(3,0)$	$(1,-4)$		1/2
$f(-x)$	$(-2,0)$ and $(1,0)$	$(0,-2)$		1/2
$\frac{1}{2}f(-x)$	$(-2,0)$ and $(1,0)$	$(0,-1)$		1/2
$f(2x)$	$(-\frac{1}{2},0)$ and $(1,0)$	$(0,-2)$		0/2
$f(-2x)$	$(-1,0)$ and $(\frac{1}{2},0)$	$(0,-2)$		0/2
$f\left(-\frac{x}{2}\right)$	$(-4,0)$ and $(2,0)$	$(0,-2)$		0/2
$-f\left(\frac{x}{2}\right)$	$(-2,0)$ and $(4,0)$	$(0,2)$		0/2
$-f\left(-\frac{x}{2}\right)$	$(-4,0)$ and $(2,0)$	$(0,2)$		0/2

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ start to differentiate •² complete differentiation •³ calculate rate of change	•¹ $4(3-2x)^3 \ldots$ •² $\ldots\times(-2)$ •³ 1000	3

Notes:

1. Correct answer with no working, award 0/3.
2. Accept $4u^3\times(-2)$ where $u=3-2x$ for •¹.
3. Where candidates evaluate $f(4)$, award 0/3, see Candidate B.
4. •³ is only available for evaluating expressions equivalent to $k(3-2x)^3$.

Commonly Observed Responses:

Candidate A $f'(x)=4(3-2x)^3\times(-2)$ •¹ ✓ •² ✓ $f'(x)=8(3-2x)^3$ $f'(4)=-1000$ •³ ✗	**Candidate B - evaluating f(x)** $f'(x)=(3-2x)^4$ •¹ ✗ •² ✗ $f'(4)=625$ •³ ✗
Candidate C - differentiating over two lines $4(3-2x)^3$ •¹ ✓ $4(3-2x)^3\times 2$ •² ✗ -1000 •³ $\boxed{\checkmark_1}$	**Candidate D - differentiating over two lines** $4(3-2x)^3$ •¹ ✓ $4(3-2x)^3\times -2$ •² ^ 1000 •³ $\boxed{\checkmark_1}$
Candidate E - insufficient evidence for mark 1 $f'(x)=8(3-2x)^3$ •¹ ✗ •² ✗ $f'(4)=-1000$ •³ $\boxed{\checkmark_1}$	**Candidate F** $4(3-2x)^3$ •¹ ✓ •² ^ -500 •³ $\boxed{\checkmark_1}$

Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1**	**Method 1**	3
			•¹ equate composite function to x	•¹ $f\left(f^{-1}(x)\right)=x$	
			•² write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$	•² $x=\dfrac{2}{f^{-1}(x)}+3$	
			•³ state inverse function	•³ $f^{-1}(x)=\dfrac{2}{x-3}$	
			Method 2	**Method 2**	
			•¹ write as $y=f(x)$ and start to rearrange	•¹ $y=f(x)\Rightarrow x=f^{-1}(y)$ $y-3=\dfrac{2}{x}$	
			•² express x in terms of y	•² $x=\dfrac{2}{y-3}$	
			•³ state inverse function	•³ $f^{-1}(y)=\dfrac{2}{y-3}$ $\Rightarrow f^{-1}(x)=\dfrac{2}{x-3}$	

Notes:

1. In Method, 1 accept $x=\dfrac{2}{f^{-1}(x)}+3$ for •¹ and •².
2. In Method 2, accept '$y-3=\dfrac{2}{x}$' without reference to $y=f(x)\Rightarrow x=f^{-1}(y)$ at •¹.
3. In Method 2, accept $f^{-1}(x)=\dfrac{2}{x-3}$ without reference to $f^{-1}(y)$ at •³.
4. In Method 2, beware of candidates with working where each line is not mathematically equivalent - see Candidates A and B for example.
5. At •³ stage, accept f^{-1} written in terms of any dummy variable eg $f^{-1}(y)=\dfrac{2}{y-3}$.
6. $y=\dfrac{2}{x-3}$ does not gain •³.
7. $f^{-1}(x)=\dfrac{2}{x-3}$ with no working gains 3/3.
8. In Method 2, where candidates make multiple algebraic errors at the •² stage, •³ is still available.

Question	Generic scheme	Illustrative scheme	Max mark
6.	(continued)		

Commonly Observed Responses:

Candidate A $f(x)=\frac{2}{x}+3$ $y=\frac{2}{x}+3$ $y-3=\frac{2}{x}$ $x=\frac{2}{y-3}$ •1 ✓ •2 ✓ $y=\frac{2}{x-3}$ •3 ✗ $f^{-1}(x)=\frac{2}{x-3}$	**Candidate B** $f(x)=\frac{2}{x}+3$ $y=\frac{2}{x}+3$ $x=\frac{2}{y}+3$ •1 ✗ $x-3=\frac{2}{y}$ $y=\frac{2}{x-3}$ •2 $\boxed{✓_1}$ $f^{-1}(x)=\frac{2}{x-3}$ •3 $\boxed{✓_1}$
Candidate C - BEWARE $f'(x)=\ldots$ •3 ✗	**Candidate D** $x\rightarrow\frac{1}{x}\rightarrow\frac{2}{x}\rightarrow\frac{2}{x}+3=f(x)$ $\times 2\rightarrow +3$ $\therefore -3\rightarrow \div 2$ •1 ✓ $\frac{2}{x-3}$ (invert) •2 ✓ $f^{-1}(x)=\frac{2}{x-3}$ •3 ✓
Candidate E ⋮ •1 ✓ •2 ✓ $f^{-1}(x)=\left(\frac{x-3}{2}\right)^{-1}$ •3 ✓	**Candidate F** ⋮ •1 ✓ •2 ✓ $f^{-1}(x)=\sqrt[-1]{\frac{x-3}{2}}$ •3 ✓
Candidate G $y=\frac{2}{x}+3$ $xy=5$ •1 ✗ $x=\frac{5}{y}$ •2 $\boxed{✓_2}$ $f^{-1}(x)=\frac{5}{x}$ •3 $\boxed{✓_1}$ **However** $f^{-1}(x)=\frac{2+3}{x}$ •3 ^	

Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ use double angle formula to express equation in terms of $\sin x°$	•¹ $\ldots = 3\left(1-2\sin^2 x°\right)$	5
			•² arrange in standard quadratic form	•² $6\sin^2 x° + \sin x° - 1 = 0$	
			•³ factorise or use quadratic formula	•³ $\left(3\sin x° - 1\right)\left(2\sin x° + 1\right)\left(=0\right)$ or $\sin x° = \dfrac{-1 \pm \sqrt{25}}{12}$	
			•⁴ solve for $\sin x°$	•⁴ $\sin x° = \dfrac{1}{3}$, •⁵ $\sin x° = -\dfrac{1}{2}$	
			•⁵ solve for x	•⁵ 19.47…, 160.52…, 210, 330	

Notes:

1. Substituting $1-2\sin^2 A$ or $1-2\sin^2\alpha$ for $\cos 2x°$ at the •¹ stage should be treated as bad form provided the equation is written in terms of x at •² stage. Otherwise, •¹ is not available.
2. Do not penalise the omission of degree signs.
3. '$=0$' must appear by •³ stage for •² to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at •² stage for •² to be awarded.
4. Candidates may express the equation obtained at •² in the form $6S^2 + S - 1 = 0$, $6x^2 + x - 1 = 0$ or using any other dummy variable at the •³ stage. In these cases, award •³ for $(3S-1)(2S+1)$ or $(3x-1)(2x+1)$.

   However, •⁴ is only available if $\sin x°$ appears explicitly at this stage - see Candidate A.
5. The equation $1 - 6\sin^2 x° - \sin x° = 0$ does not gain •² unless •³ has been awarded.
6. •³ is awarded for identifying the factors of the quadratic obtained at •² eg "$3\sin x° - 1 = 0$ and $2\sin x° + 1 = 0$".
7. •⁴ and •⁵ are only available as a consequence of trying to solve a quadratic equation - see Candidate B.
8. •³, •⁴ and •⁵ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$ - see Candidate C.
9. •⁵ is only available where at least one of the equations at •⁴ leads to two solutions for x.
10. Do not penalise additional (correct) solutions at •⁵. However see Candidates E and F.
11. Accept answers which round to 19, 19.5 and 161.

Question	Generic scheme	Illustrative scheme	Max mark
7.	(continued)		

Commonly Observed Responses:

Candidate A

$\vdots$	$\bullet^1$ ✓ $\bullet^2$ ✓
$6S^2+S-1=0$	
$(3S-1)(2S+1)=0$	$\bullet^3$ ✓
$S=\frac{1}{3},\quad S=-\frac{1}{2}$	$\bullet^4$ ^
$x=19.5,\ 160.5,\ 210,\ 330$	$\bullet^5$ $\boxed{\checkmark_1}$

Candidate B - not solving a quadratic

$\vdots$	$\bullet^1$ ✓
$6\sin^2 x^\circ+\sin x^\circ-1=0$	$\bullet^2$ ✓
$7\sin x^\circ-1=0$	$\bullet^3$ ✗
$\sin x^\circ=\frac{1}{7}$	$\bullet^4$ $\boxed{\checkmark_2}$
$x=8.2$	$\bullet^5$ $\boxed{\checkmark_2}$

Candidate C - not in standard quadratic form

$\sin x^\circ+2=3-6\sin^2 x^\circ$	$\bullet^1$ ✓
$6\sin^2 x^\circ+\sin x^\circ=1$	$\bullet^2$ $\boxed{\checkmark_2}$
$\sin x^\circ(6\sin x^\circ+1)=1$	$\bullet^3$ $\boxed{\checkmark_2}$
$\sin x^\circ=1 \qquad 6\sin x^\circ+5=1$	
$\Rightarrow \sin x=-\frac{4}{6}$	$\bullet^4$ ✗
$90,\ 221.8,\ 318.2$	$\bullet^5$ ✗

Candidate D - reading $\cos 2x^\circ$ as $\cos^2 x^\circ$

$\sin x^\circ+2=3\cos^2 x^\circ$	$\bullet^1$ ✗
$\sin x^\circ+2=3\left(1-\sin^2 x^\circ\right)$	
$3\sin^2 x^\circ+\sin x^\circ-1=0$	$\bullet^2$ $\boxed{\checkmark_1}$
$\sin x^\circ=\frac{-1\pm\sqrt{13}}{6}$	$\bullet^3$ $\boxed{\checkmark_1}$
$\sin x^\circ=0.434\ldots,\quad \sin x^\circ=-0.767\ldots$	$\bullet^4$ $\boxed{\checkmark_1}$
$25.7,\ 154.3,\ 230.1,\ 309.9$	$\bullet^5$ $\boxed{\checkmark_1}$

Candidate E

$\vdots$	$\bullet^1$ ✓ $\bullet^2$ ✓
$(3\sin x^\circ-1)(2\sin x^\circ+1)=0$	$\bullet^3$ ✓
$\sin x^\circ=\frac{1}{3},\quad \sin x^\circ=-\frac{1}{2}$	$\bullet^4$ ✓
$x=19,\ x=161 \qquad x=30$	
$x=210,\ x=330$	$\bullet^5$ ✗

However, where the final solution(s) are clearly identified by the candidate award $\bullet^5$

Candidate F

$\vdots$	$\bullet^1$ ✓ $\bullet^2$ ✓
$(3\sin x^\circ-1)(2\sin x^\circ+1)=0$	$\bullet^3$ ✓
$\sin x^\circ=\frac{1}{3},\quad \sin x^\circ=-\frac{1}{2}$	$\bullet^4$ ✓
$x=19,\ 161,\ 30,\ 210,\ 330$	$\bullet^5$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
8.			**Method 1**	**Method 1**	**5**
			•¹ integrate using "upper - lower"	•¹ $\int\left(\left(x^3-2x^2-4x+1\right)-\left(x-5\right)\right)dx$	
			•² identify limits	•² $\int_{-2}^{1}\left(\left(x^3-2x^2-4x+1\right)-\left(x-5\right)\right)dx$	
			•³ integrate	•³ $\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$	
			•⁴ substitute limits	•⁴ $\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{5(1)^2}{2}+6(1)\right)-\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{5(-2)^2}{2}+6(-2)\right)$	
			•⁵ calculate shaded area	•⁵ $\frac{63}{4}$ or $15\frac{3}{4}$	
			Method 2	**Method 2**	**5**
			•¹ know to integrate between appropriate limits for both integrals	•¹ $\int_{-2}^{1}\ldots dx$ and $\int_{-2}^{1}\ldots dx$	
			•² integrate both functions	•² $\frac{x^4}{4}-\frac{2x^3}{3}-\frac{4x^2}{2}+x$ and $\frac{x^2}{2}-5x$	
			•³ substitute limits into both expressions	•³ $\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{4(1)^2}{2}+(1)\right)-\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{4(-2)^2}{2}+(-2)\right)$ **and** $\left(\frac{(1)^2}{2}-5(1)\right)-\left(\frac{(-2)^2}{2}-5(-2)\right)$	
			•⁴ evaluate both integrals	•⁴ $-\frac{3}{4}$ and $-\frac{33}{2}$	
			•⁵ evidence of subtracting areas	•⁵ $-\frac{3}{4}-\left(-\frac{33}{2}\right)=\frac{63}{4}$	

Question		Generic scheme	Illustrative scheme	Max mark
8.	(continued)			

Notes:
1. Correct answer with no working – award 1/5. 2. In Method 1, treat the absence of brackets at $\bullet^1$ stage as bad form only if the correct integral is obtained at $\bullet^3$ - see Candidates A and B. 3. Do not penalise lack of 'dx' at $\bullet^1$. 4. Limits and 'dx' must appear by the $\bullet^2$ stage for $\bullet^2$ to be awarded in Method 1 and by the $\bullet^1$ stage for $\bullet^1$ to be awarded in Method 2. 5. Where a candidate differentiates one or more terms at $\bullet^3$, then $\bullet^3$, $\bullet^4$ and $\bullet^5$ are unavailable. 6. Accept unsimplified expressions at $\bullet^3$ e.g. $\frac{x^4}{4}-\frac{2x^3}{3}-\frac{4x^2}{2}+x-\frac{x^2}{2}+5x$. 7. Do not penalise the inclusion of '$+c$'. 8. Do not penalise the continued appearance of the integral sign after $\bullet^2$ 9. Candidates who substitute limits without integrating do not gain $\bullet^3$, $\bullet^4$ or $\bullet^5$. 10. $\bullet^5$ is not available where solutions include statements such as '$-\frac{63}{4}=\frac{63}{4}$ square units' - see Candidate B. 11. Where a candidate only integrates x^3-2x^2-4x+1 or another cubic or quartic expression, only $\bullet^3$ and $\bullet^4$ are available (from Method 1).

Question	Generic scheme	Illustrative scheme	Max mark
8. (continued)			

Commonly Observed Responses:

Candidate A - bad form corrected

$\int_{-2}^{1} x^3-2x^2-4x+1-x-5\,dx$ $\quad \bullet^2$ ✓

$=\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$ $\quad \bullet^3$ ✓ $\Rightarrow \bullet^1$ ✓

Bad form at $\bullet^1$ must be corrected by the integration stage and may also take the form of a missing minus sign

Candidate B

$\int_{-2}^{1} x^3-2x^2-4x+1-x-5\,dx$ $\quad \bullet^1$ ✗ $\bullet^2$ ✓

$=\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}-4x$ $\quad \bullet^3$ $\boxed{\checkmark_1}$

$=\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{5(1)^2}{2}-4(1)\right)$

$-\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{5(-2)^2}{2}-4(-2)\right)$ $\quad \bullet^4$ $\boxed{\checkmark_1}$

$-\frac{57}{4}$ cannot be negative so $=\frac{57}{4}$ $\quad \bullet^5$ ✗

However, $\int \ldots = -\frac{57}{4}$ so Area $=\frac{57}{4}$ $\quad \bullet^5$ $\boxed{\checkmark_1}$

Candidate C - lower – upper

$\int_{-2}^{1} \left((x-5)-(x^3-2x^2-4x+1)\right)dx$ $\quad \bullet^2$ ✓

$-\frac{x^4}{4}+\frac{2x^3}{3}+\frac{5x^2}{2}-6x$ $\quad \bullet^3$ ✓

$\left(-\frac{(1)^4}{4}+\frac{2(1)^3}{3}+\frac{5(1)^2}{2}-6(1)\right)-$

$\left(-\frac{(-2)^4}{4}+\frac{2(-2)^3}{3}+\frac{5(-2)^2}{2}-6(-2)\right)$ $\quad \bullet^4$ ✓

$-\frac{63}{4}$

So Area $=\frac{63}{4}$ $\quad \bullet^1$ ✓ $\bullet^5$ ✓

Candidate D - reversed limits

$\int_{1}^{-2} \left((x^3-2x^2-4x+1)-(x-5)\right)dx$ $\quad \bullet^1$ ✓

$\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$ $\quad \bullet^3$ ✓

$\left(\frac{(-2)^4}{4}-\frac{2(-2)^3}{3}-\frac{5(-2)^2}{2}+6(-2)\right)$

$-\left(\frac{(1)^4}{4}-\frac{2(1)^3}{3}-\frac{5(1)^2}{2}+6(1)\right)$ $\quad \bullet^4$ ✓

$-\frac{63}{4}$

So Area $=\frac{63}{4}$ $\quad \bullet^2$ ✓ $\bullet^5$ ✓

Candidate E - 'upper' – 'lower'

$=x^3-2x^2-5x+6$

$\int_{-2}^{1} \left(x^3-2x^2-5x+6\right)dx$ $\quad \bullet^1$ ✓ $\bullet^2$ ✓

$\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x$ $\quad \bullet^3$ ✓

$\frac{37}{12}-\left(-\frac{38}{3}\right)$ $\quad \bullet^4$ ✓

$\frac{63}{4}$ $\quad \bullet^5$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] use compound angle formula •[2] compare coefficients •[3] process for k •[4] process for a and express in required form	•[1] $k\sin x^\circ\cos a^\circ + k\cos x^\circ\sin a^\circ$ **stated explicitly** •[2] $k\cos a^\circ = -3,\ k\sin a^\circ = 7$ **stated explicitly** •[3] $\sqrt{58}$ •[4] $\sqrt{58}\sin\left(x + 113.19\ldots\right)^\circ$.	**4**

Notes:

1. Do not penalise the omission of degree symbols in this question.
2. Accept $k\left(\sin x^\circ\cos a^\circ + \cos x^\circ\sin a^\circ\right)$ at •[1].
3. Treat $k\sin x^\circ\cos a^\circ + \cos x^\circ\sin a^\circ$ as bad form only if the equations at the •[2] stage both contain k.
4. $\sqrt{58}\sin x^\circ\cos a^\circ + \sqrt{58}\cos x^\circ\sin a^\circ$ or $\sqrt{58}\left(\sin x^\circ\cos a^\circ + \cos x^\circ\sin a^\circ\right)$ are acceptable for •[1] and •[3].
5. •[2] is not available for $k\cos x^\circ = -3$ and $k\sin x^\circ = 7$, however •[4] may still be gained - see Candidate E.
6. •[3] is only available for a single value of k, $k > 0$.
7. •[4] is not available for a value of a given in radians.
8. Accept values of a which round to 113.
9. Candidates may use any form of the wave function for •[1], •[2] and •[3]. However, •[4] is only available if the wave is interpreted in the form $k\sin\left(x + a\right)^\circ$.
10. Evidence for •[4] may appear in part (b).

Question	Generic scheme	Illustrative scheme	Max mark
9.	(continued)		

Commonly Observed Responses:

Candidate A $\bullet^1$ ^ $\sqrt{58}\cos a^\circ = -3$ $\sqrt{58}\sin a^\circ = 7$ $\bullet^2$ ✓ $\bullet^3$ ✓ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\ldots$ $\sqrt{58}\sin\left(x + 113.19\ldots\right)^\circ$ $\bullet^4$ ✓	**Candidate B** $k\sin x^\circ\cos a^\circ + k\cos x^\circ\sin a^\circ$ $\bullet^1$ ✓ $\cos a^\circ = -3$ $\sin a^\circ = 7$ $\bullet^2$ ✗ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\ldots$ Not consistent with equations at $\bullet^2$. $\sqrt{58}\sin\left(x + 113.19\ldots\right)^\circ$ $\bullet^3$ ✓ $\bullet^4$ ✗	**Candidate C** $\sin x^\circ\cos a^\circ + \cos x^\circ\sin a^\circ$ $\bullet^1$ ✗ $\cos a^\circ = -3$ $\sin a^\circ = 7$ $\bullet^2$ $\boxed{✓_2}$ $k = \sqrt{58}$ $\bullet^3$ ✓ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\ldots$ $\sqrt{58}\sin\left(x + 113.19\ldots\right)^\circ$ $\bullet^4$ ✗
Candidate D - errors at $\bullet^2$ $k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓ $k\cos a^\circ = 7$ $k\sin a^\circ = -3$ $\bullet^2$ ✗ $\tan a^\circ = -\frac{3}{7}$ $a = 336.80\ldots$ $\sqrt{58}\sin\left(x + 336.80\ldots\right)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓_1}$	**Candidate E - use of x at $\bullet^2$** $k\sin x\cos a + k\cos x\sin a$ $\bullet^1$ ✓ $k\cos x^\circ = -3$ $k\sin x^\circ = 7$ $\bullet^2$ ✗ $\tan a^\circ = -\frac{7}{3}$ $a = 113.19\ldots$ $\sqrt{58}\sin\left(x + 113.19\ldots\right)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓_1}$	**Candidate F** $k\sin \mathrm{A}\cos \mathrm{B} + k\cos \mathrm{A}\sin \mathrm{B}$ $\bullet^1$ ✗ $k\cos \mathrm{A} = -3$ $k\sin \mathrm{A} = 7$ $\bullet^2$ ✗ $\tan \mathrm{A} = -\frac{7}{3}$ $\mathrm{A} = 113.19\ldots$ $\sqrt{58}\sin\left(x + 113.19\ldots\right)^\circ$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{✓_1}$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(i)	•5 state maximum value	•5 $2\sqrt{58}$	1
		(ii)	**Method 1** •6 start to solve •7 state value of x **Method 2** •6 start to solve •7 state value of x	**Method 1** •6 $x+113.19...=90$ leading to $x=-23.19...$ •7 $x=336.80...$ **Method 2** •6 $x+113.19...=450$ •7 $x=336.80...$	2

Notes:

11. •7 is only available where an angle outwith the range $0\le x<360$ needs to be considered - see Candidate G.
12. •7 is only available where •6 has been awarded. However, see Candidate K.

Commonly Observed Responses:

Candidate G - not considering angle outwith $0\le x<360$ $\sqrt{58}\sin(x-23)°$ from part (a) $x-23=90$ •6 ✓1 $x=113$ •7 ✓2	**Candidate H - simplification** (i) $2\sqrt{58}$ •5 ✓ (ii) $\sqrt{58}\sin(x+113)°=\sqrt{58}$ $x+113=90$ $x=-23$ •6 ✓ $x=337$ •7 ✓
Candidate I - follow-through marking (i) $\sqrt{58}$ •5 ✗ (ii) $2\sqrt{58}\sin(x+113)°=\sqrt{58}$ $x+113=30$ $x=-83$ •6 ✓1 $x=277$ •7 ✓1	**Candidate J - graphical approach** (i) $\sqrt{58}$ •5 ✗ (ii) max occurs when $x+113=90$ $x=-23$ •6 ✓ $x=337$ •7 ✓
Candidate K - no acknowledgement of $\times 2$ (i) $\sqrt{58}$ •5 ✗ (ii) $\sqrt{58}\sin(x+113)°=\sqrt{58}$ $x+113=90$ $x=-23$ •6 ✗ $x=337$ •7 ✓1	

Question	Generic scheme	Illustrative scheme	Max mark
10.	**Method 1**	**Method 1**	4
	•¹ differentiate one term	•¹ $6x^2\ldots$ or $+18x\ldots$ or -24	
	•² complete differentiation and interpret condition	•² $6x^2+18x-24<0$	
	•³ determine zeros of quadratic expression	•³ 1 and -4	
	•⁴ state range with justification	•⁴ $-4<x<1$ with eg labelled sketch	
	Method 2	**Method 2**	4
	•¹ differentiate one term	•¹ $6x^2\ldots$ or $+18x\ldots$ or -24	
	•² complete differentiation and determine zeros of quadratic expression	•² $6x^2+18x-24$ **and** 1 and -4	
	•³ construct nature table(s)	•³ $\begin{array}{c\|c\|c\|c\|c\|c} x & \ldots & -4 & \ldots & 1 & \ldots \\ \hline f'(x) & + & 0 & - & 0 & + \end{array}$	
	•⁴ interpret sign of derivative and state range	•⁴ decreasing when $f'(x)<0$ so $-4<x<1$	

Notes:

1. At •³ do not penalise candidates who fail to extract the common factor or who have divided the quadratic inequality by 6.
2. •³ and •⁴ are not available to candidates who arrive at a linear expression at •².
3. Accept the appearance of -4 and 1 within inequalities for •³.
4. At •⁴, accept "$x>-4$, $x<1$" together with the required justification.

Commonly Observed Responses:

Candidate A

$6x^2+18x-24<0$ •¹ ✓ •² ✓

$6x^2+18x-24=0$

$x=-4, 1$ •³ ✓

$-4<x<1$ with sketch •⁴ ✓

Candidate B - no initial inequation

$6x^2+18x-24=0$ •¹ ✓ •² ✗

$x=-4, 1$ •³ ✓

$-4<x<1$ with sketch •⁴ ✗

Candidate C

Decreasing when $f'(x)<0$

$f'(x)=6x^2+18x-24$ •¹ ✓ •² ✓

⋮

Candidate D - condition applied after simplification

$f'(x)=6x^2+18x-24$ •¹ ✓

$x^2+3x-4<0$ •² ^

$x=-4, 1$ •³ ✓

$-4<x<1$ with sketch •⁴ ✓

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>11.</td><td>(a)</td><td></td><td>•1 state centre of C_1

•2 state centre of C_2

•3 calculate distance between centres</td><td>•1 $(4, -2)$

•2 $(-1, 3)$

•3 $\sqrt{50}$ or $5\sqrt{2}$ or 7.07…</td><td>3</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. Accept $x=4,\ y=-2$ for •1 and $x=-1,\ y=3$ •2. Do not accept $g=1,\ f=-3$ for •2.
2. Do not penalise lack of brackets in •1 and •2.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td></td><td>(b)</td><td></td><td>•4 state radius of C_1

•5 calculate radius of C_2

•6 demonstrate and communicate result</td><td>•4 $r_1=\sqrt{37}$ or 6.08…

•5 $r_2=\sqrt{17}$ or 4.12…

•6 $10.20\ldots > 7.07\ldots\ (>1.95\ldots)$
$\therefore$ circles intersect at two distinct points</td><td>3</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">3. Accept $\sqrt{1^2+3^2+7}=\sqrt{17}$ or $\sqrt{1^2+-3^2+7}=\sqrt{17}$ for •5. However, do not accept $\sqrt{(-1)^2+3^2+7}=\sqrt{17}$.
4. At •6 comparison must be made using decimals. Do not accept $\sqrt{37}+\sqrt{17}>\sqrt{50}$ without any further working.
5. Evidence for •4 and •5 may be found in part (a).
6. For candidates who use simultaneous equations, award •4 for substitution of $y=x+1$ into the equation of one of the circles, •5 for rearranging in standard quadratic form and •6 for obtaining distinct x-coordinates.
7. Do not penalise the omission of “at two distinct points” at •6.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="6"></td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ integrate one term	•¹ eg $\frac{8x^4}{4}\ldots$	**4**
			•² complete integration	•² eg $\ldots+3x+c$	
			•³ substitute for x and y	•³ $3=\frac{8\times(-1)^4}{4}+3\times(-1)+c$	
			•⁴ state expression for y	•⁴ $y=2x^4+3x+4$	

Notes:

1. For candidates who omit $+c$ only •¹ is available.
2. For candidates who differentiate either term, •², •³, and •⁴ are not available.
3. Do not penalise the appearance of an integral sign and/or dx at •² and •³.

Commonly Observed Responses:

Candidate A - incomplete substitution		**Candidate B - partial integration**	
$y=2x^4+3x+c$	•¹ ✓ •² ✓	$y=2x^4+3+c$	•¹ ✓ •² ✗
$y=2(-1)^4+3(-1)+c$		$3=2(-1)^4+3+c$	•³ $\boxed{✓_1}$
$c=4$	•³ ^	$c=-2$	
$y=2x^4+3x+4$	•⁴ $\boxed{✓_1}$	$y=2x^4+1$	•⁴ $\boxed{✓_1}$
Candidate C - integrating over two lines			
$y=2x^4+3x$	•¹ ✓ •² ✗		
$y=2x^4+3x+c$			
$3=2(-1)^4+3(-1)+c$	•³ ✓		
$y=2x^4+3x+4$	•⁴ ✓		

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] calculate concentration	•[1] 9.38... (mg/l)	**1**

Notes:

1. Accept any answer which rounds to 9.4 for •[1].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[2] substitute •[3] write in logarithmic form •[4] process for t	•[2] $0.66 = 11 \times e^{-0.0053t}$ •[3] $\log_e \frac{0.66}{11} = -0.0053t$ •[4] 530.83... (minutes)	**3**

Notes:

2. Where values other than 0.66 are used in the substitution, •[3] and •[4] are still available.
3. Evidence for •[3] must be stated explicitly.
4. At •[3] all exponentials must be processed.
5. Any base may be used at •[3] stage - see Candidate A.
6. Accept $\ln 0.06 = -0.0053t \ln e$ for •[3].
7. Accept any answer where $530 \leq t \leq 532$ at •[4].
8. •[4] is unavailable where a candidate rounds the value of $\ln 0.06$ to fewer than 2 decimal places.
9. The calculation at •[4] must follow from the valid use of exponentials and logarithms at •[2] and •[3].
10. For candidates with no working or who take an iterative approach to arrive at $t = 532$, $t = 531$ or $t = 530$ award 1/3. However, if, in any iterations C_t is evaluated for $t = 530$ and $t = 531$ leading to a final answer of $t = 531$ (minutes) then award 3/3.

Commonly Observed Responses:

Candidate A		Candidate B	
$0.66 = 11e^{-0.0053t}$	•[2] ✓	$0.66 = 11e^{-0.0053t}$	•[2] ✓
$0.06 = e^{-0.0053t}$		$t = 531$ minutes	•[3] ^ •[4] $\boxed{✓_1}$
$\log_{10} 0.06 = -0.0053t \log_{10} e$	•[3] ✓		
$t = 531$ minutes	•[4] ✓		

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)	(i)	•¹ express A in terms of x and h	•¹ $(A=)6x^2+10xh$	**1**
		(ii)	•² express h in terms of x •³ substitute for h and demonstrate result	•² $h=\dfrac{7200-6x^2}{10x}$ •³ $V=3x\times2x\times\left(\dfrac{7200-6x^2}{10x}\right)$ leading to $V=4320x-\dfrac{18}{5}x^3$	**2**

Notes:

1. Accept unsimplified expressions for •¹.
2. •² is only available where the (simplified) expression for A contains at least 2 terms.
3. The substitution for h at •³ must be clearly shown for •³ to be awarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁴ differentiate •⁵ equate expression for derivative to 0 •⁶ solve for x •⁷ verify nature	•⁴ $4320-\dfrac{54}{5}x^2$ •⁵ $4320-\dfrac{54}{5}x^2=0$ •⁶ 20 •⁷ table of signs for a derivative $\begin{array}{r\|c\|c\|c\|} x & \dots & 20 & \dots \\ \hline V'(x) & + & 0 & - \\ \text{shape} & / & — & \backslash \end{array}$ $\therefore$ maximum (when $x=20$)	**4**

Notes:

4. For any approach which does not use differentiation award 0/4.
5. •⁵ can be awarded for $\dfrac{54}{5}x^2=4320$.
6. For candidates who integrate any term at the •⁴ stage, only •⁵ is available on follow through for setting their 'derivative' to 0.
7. Ignore the appearance of −20 at mark •⁶.
8. Where −20 is considered in a nature table (or second derivative), "$x=20$" must be clearly identified as leading to the maximum area.
9. •⁶ and •⁷ are not available to candidates who state that the maximum exists at a negative value of x.
10. Do not penalise statements such as "max volume is 20" or "max is 20" at •⁷.

Question	Generic scheme	Illustrative scheme	Max mark
14.	(continued)		

Commonly Observed Responses:

Candidate A - second derivative	Candidate B - beware of multiplying before equating
$V''(x) = -\frac{108}{5}x$ $V''(20) < 0 \;\therefore$ maximum •[7] ✓	$V'(x) = 4320 - \frac{54}{5}x^2$ •[4] ✓ $V'(x) = 21600 - 54x^2$ $21600 - 54x^2 = 0$ •[5] ✗ $x = 20$ •[6] ✓[1]
Candidate C Stationary points when $V'(x) = 0$ $V'(x) = 4320 - \frac{54}{5}x^2$ •[4] ✓ •[5] ✓	

For the table of signs for a derivative, accept:

x	20^-	20	20^+
$V'(x)$	$+$	0	$-$
Slope or shape	/	—	\

x	$\rightarrow$	20	$\rightarrow$
$V'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Arrows are taken to mean 'in the neighbourhood of'

x	a	20	b
$V'(x)$	$+$	0	$-$
Slope or shape	/	—	\

Where $a < 20$ and $b > 20$

For the table of signs for a derivative, accept:

x	$\rightarrow$	-20	$\rightarrow$	20	$\rightarrow$
$V'(x)$	$-$	0	$+$	0	$-$
Slope or shape	\	—	/	—	\

Since the function is continuous $-20 \rightarrow 20$ is acceptable

x	a	-20	b	20	c
$V'(x)$	$-$	0	$+$	0	$-$
Slope or shape	\	—	/	—	\

Since the function is continuous $-20 < b < 20$ is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $V'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $V'(x)$ are: V', $\frac{dV}{dx}$, and $4320 - \frac{54}{5}x^2$. Accept $\frac{dy}{dx}$ only where candidates have previously used $y = 4320x - \frac{18}{5}x^3$ in their working.

Question			Generic scheme	Illustrative scheme	Max mark
15.			$\bullet^1$ determine gradient of tangent	$\bullet^1$ $-\frac{1}{3}$	**4**
			$\bullet^2$ determine gradient of radius	$\bullet^2$ 3	
			$\bullet^3$ strategy to find centre	$\bullet^3$ eg $y=3x-1$ or $3=\dfrac{y-5}{x-2}$	
			$\bullet^4$ state coordinates of centre	$\bullet^4$ $(0,-1)$	

Notes:

1. Ignore errors in processing the constant term in $\bullet^1$.
2. Do not accept $m=-\frac{1}{3}x$ for $\bullet^1$. However $\bullet^2$, $\bullet^3$ and $\bullet^4$ are still available where the candidate uses a numerical value for $m_{\perp}$.
3. Accept $y-5=3(x-2)$ as evidence for $\bullet^3$.
4. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient along with a point on the y-axis.
5. Where candidates use "stepping out" with the perpendicular gradient, the diagram must be consistent with the solution to gain $\bullet^3$ and $\bullet^4$.
6. Accept "$x=0$", "$y=-1$" stated explicitly for $\bullet^4$.

Commonly Observed Responses:

Candidate A - perpendicular gradient clearly stated

$x+3y=17$

$m_{\perp}=3$ — $\bullet^1$ ✓ $\bullet^2$ ✓

$y=3x-1$ — $\bullet^3$ ✓

Candidate B - no communication for perpendicular gradient

$x+3y=17$

$y=-\frac{1}{3}x+\frac{17}{3}$

$m=3$ — $\bullet^1$ ^ $\bullet^2$ $\boxed{\checkmark_1}$

$y=3x-1$ — $\bullet^3$ $\boxed{\checkmark_1}$

$\bullet^4$ is available

Candidate C - no communication for perpendicular gradient or rearrangement

$x+3y=17$

$m=3$ — $\bullet^1$ ^ $\bullet^2$ ^

$y=3x-1$ — $\bullet^3$ $\boxed{\checkmark_2}$

$\bullet^4$ is not available

Candidate D - using geometry

⋮ — $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

Using point diametrically opposite (2,5), by symmetry identify that x-coordinate is −2.

$\therefore y=3(-2)-1=-7$.

Centre is midpoint of (−2,−7) and (2,5).

$\therefore$ centre is (0,−1) — $\bullet^4$ ✓

Candidate E - incorrect gradient

$x+3y=17$

$3y=-x+17$

$m_{\perp}=1$ — $\bullet^1$ ^ $\bullet^2$ ✗

$1=\dfrac{5-y}{2-0}$ — $\bullet^3$ $\boxed{\checkmark_1}$

Centre is at $(0,3)$ — $\bullet^4$ $\boxed{\checkmark_1}$

[END OF MARKING INSTRUCTIONS]