X100/701

NATIONAL
QUALIFICATIONS
2008

TUESDAY, 20 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**

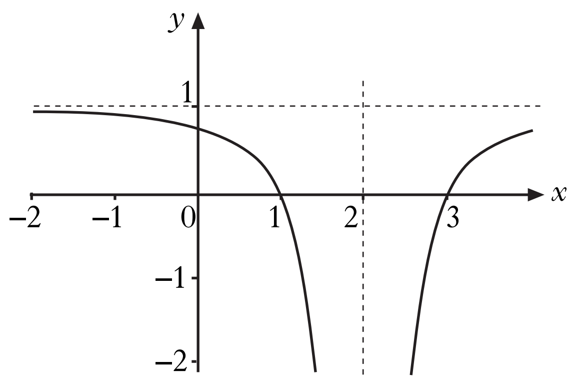
Answer all the questions.

1. The first term of an arithmetic sequence is 2 and the 20th term is 97. Obtain the sum of the first 50 terms. **4**

2. (*a*) Differentiate $f(x) = \cos^{-1}(3x)$ where $-\frac{1}{3} < x < \frac{1}{3}$. **2**

(*b*) Given $x = 2\sec\theta$, $y = 3\sin\theta$, use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ. **3**

3. Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph $y = -f(x + 1)$ showing its asymptotes. Write down the equations of the asymptotes. **4**



4. Express $\dfrac{12x^2 + 20}{x(x^2 + 5)}$ in partial fractions. **3**

Hence evaluate

$$\int_1^2 \frac{12x^2 + 20}{x(x^2 + 5)}\,dx.$$ **3**

5. A curve is defined by the equation $xy^2 + 3x^2y = 4$ for $x > 0$ and $y > 0$.

Use implicit differentiation to find $\frac{dy}{dx}$. **3**

Hence find an equation of the tangent to the curve where $x = 1$. **3**

Marks

6. Let the matrix $A = \begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$.

(*a*) Obtain the value(s) of x for which A is singular. **2**

(*b*) When $x = 2$, show that $A^2 = pA$ for some constant p.
Determine the value of q such that $A^4 = qA$. **3**

7. Use integration by parts to obtain $\int 8x^2 \sin 4x \, dx$. **5**

8. Write down and simplify the general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{10}$. **3**

Hence, or otherwise, obtain the term in x^{14}. **2**

9. Write down the derivative of $\tan x$. **1**

Show that $1 + \tan^2 x = \sec^2 x$. **1**

Hence obtain $\int \tan^2 x \, dx$. **2**

10. A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time t.

(*a*) Obtain the value of its acceleration when $t = 0$. **1**

(*b*) At time $t = 0$, the body is at the origin O. Obtain a formula for the displacement of the body at time t. **2**

Show that the body returns to O, and obtain the time, T, when this happens. **2**

11. For each of the following statements, decide whether it is true or false and prove your conclusion.

A For all natural numbers m, if m^2 is divisible by 4 then m is divisible by 4.

B The cube of any odd integer p plus the square of any even integer q is always odd. **5**

12. Obtain the first three non-zero terms in the Maclaurin expansion of $x \ln(2 + x)$. **3**

Hence, or otherwise, deduce the first three non-zero terms in the Maclaurin expansion of $x \ln(2 - x)$. **2**

Hence obtain the first **two** non-zero terms in the Maclaurin expansion of $x \ln(4 - x^2)$. **2**

[*Throughout this question, it can be assumed that* $-2 < x < 2$.]

[Turn over for Questions 13 to 16 on *Page four*

13. Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2.$$ **7**

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$, when $x = 0$, find the particular solution. **3**

14. (*a*) Find an equation of the plane π_1 through the points $A(1, 1, 1)$, $B(2, -1, 1)$ and $C(0, 3, 3)$. **3**

(*b*) The plane π_2 has equation $x + 3y - z = 2$.

Given that the point $(0, a, b)$ lies on both the planes π_1 and π_2, find the values of a and b. Hence find an equation of the line of intersection of the planes π_1 and π_2. **4**

(*c*) Find the size of the acute angle between the planes π_1 and π_2. **3**

15. Let $f(x) = \frac{x}{\ln x}$ for $x > 1$.

(*a*) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers. **2,2**

(*b*) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$. **3**

(*c*) Obtain the coordinates of the point of inflexion. **2**

16. Given $z = \cos\theta + i\sin\theta$, use de Moivre's theorem to write down an expression for z^k in terms of θ, where k is a positive integer.

Hence show that $\frac{1}{z^k} = \cos k\theta - i\sin k\theta$. **3**

Deduce expressions for $\cos k\theta$ and $\sin k\theta$ in terms of z. **2**

Show that $\cos^2\theta\,\sin^2\theta = -\frac{1}{16}\left(z^2 - \frac{1}{z^2}\right)^2$. **3**

Hence show that $\cos^2\theta\,\sin^2\theta = a + b\cos 4\theta$, for suitable constants a and b. **2**

[*END OF QUESTION PAPER*]