

# X100/701

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NATIONAL  
QUALIFICATIONS  
2004

FRIDAY, 21 MAY  
1.00 PM - 4.00 PM

MATHEMATICS  
ADVANCED HIGHER

**Read carefully**

1. Calculators may be used in this paper.
2. Candidates should answer all questions.
3. **Full credit will be given only where the solution contains appropriate working.**



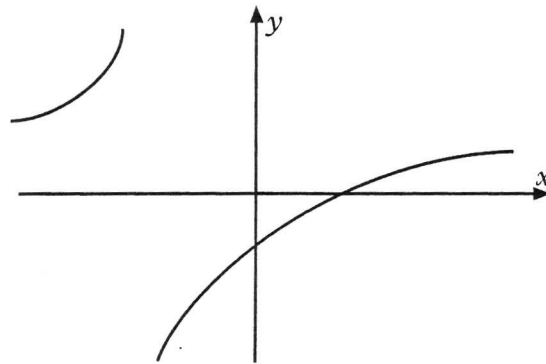
## Answer all the questions.

1. (a) Given  $f(x) = \cos^2 x e^{\tan x}$ ,  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , obtain  $f'(x)$  and evaluate  $f'(\frac{\pi}{4})$ . 3,1  
 (b) Differentiate  $g(x) = \frac{\tan^{-1} 2x}{1 + 4x^2}$ . 3
2. Obtain the binomial expansion of  $(a^2 - 3)^4$ . 3
3. A curve is defined by the equations  

$$x = 5 \cos \theta, \quad y = 5 \sin \theta, \quad (0 \leq \theta < 2\pi).$$
 Use parametric differentiation to find  $\frac{dy}{dx}$  in terms of  $\theta$ . 2  
 Find the equation of the tangent to the curve at the point where  $\theta = \frac{\pi}{4}$ . 3
4. Given  $z = 1 + 2i$ , express  $z^2(z + 3)$  in the form  $a + ib$ . 2  
 Hence, or otherwise, verify that  $1 + 2i$  is a root of the equation  

$$z^3 + 3z^2 - 5z + 25 = 0.$$
 2  
 Obtain the other roots of this equation. 2
5. Express  $\frac{1}{x^2 - x - 6}$  in partial fractions. 2  
 Evaluate  $\int_0^1 \frac{1}{x^2 - x - 6} dx$ . 4
6. Write down the  $2 \times 2$  matrix  $M_1$  associated with an anti-clockwise rotation of  $\frac{\pi}{2}$  radians about the origin. 2  
 Write down the matrix  $M_2$  associated with reflection in the  $x$ -axis. 1  
 Evaluate  $M_2 M_1$  and describe geometrically the effect of the transformation represented by  $M_2 M_1$ . 2
7. Obtain the first three non-zero terms in the Maclaurin expansion of  $f(x) = e^x \sin x$ . 5
8. Use the Euclidean algorithm to show that  $(231, 17) = 1$  where  $(a, b)$  denotes the highest common factor of  $a$  and  $b$ .  
 Hence find integers  $x$  and  $y$  such that  $231x + 17y = 1$ . 4
9. Use the substitution  $x = (u - 1)^2$  to obtain  $\int \frac{1}{(1 + \sqrt{x})^3} dx$ . 5

10. Determine whether the function  $f(x) = x^+ \sin 2x$  is odd, even or neither.  
Justify your answer. 3
11. A solid is formed by rotating the curve  $y = e^{-2x}$  between  $x = 0$  and  $x = 1$  through  $360^\circ$  about the  $x$ -axis. Calculate the volume of the solid that is formed. 5
12. Prove by induction that  $\frac{d^n}{dx^n} (xe^x) = (x+n)e^x$  for all integers  $n \geq 1$ . 5
13. The function  $f$  is defined by  $f(x) = \frac{x-3}{x+2}$ ,  $x \neq -2$ , and the diagram shows part of its graph.



- (a) Obtain algebraically the asymptotes of the graph of  $f$ . 3
- (b) Prove that  $f$  has no stationary values. 2
- (c) Does the graph of  $f$  have any points of inflexion? Justify your answer. 2
- (d) Sketch the graph of the inverse function,  $f^{-1}$ . State the asymptotes and domain of  $f^{-1}$ . 3
14. (a) Find an equation of the plane  $\pi_1$  containing the points  $A(1, 0, 3)$ ,  $B(0, 2, -1)$  and  $C(1, 1, 0)$ . 4
- Calculate the size of the acute angle between  $\pi_1$  and the plane  $\pi_2$  with equation  $x + y - z = 0$ . 3
- (b) Find the point of intersection of plane  $\pi_2$  and the line
- $$\frac{x-11}{4} = \frac{y-15}{5} = \frac{z-12}{2}.$$
- 3

[Turn over for Questions 15 and 16 on Page four

15. (a) A mathematical biologist believes that the differential equation  $x \frac{dy}{dx} - 3y = x^4$  models a process. Find the general solution of the differential equation. 5
- Given that  $y = 2$  when  $x = 1$ , find the particular solution, expressing  $y$  in terms of  $x$ . 2
- (b) The biologist subsequently decides that a better model is given by the equation  $y \frac{dy}{dx} - 3x = x^4$ .
- Given that  $y = 2$  when  $x = 1$ , obtain  $y$  in terms of  $x$ . 4
16. (a) Obtain the sum of the series  $8 + 11 + 14 + \dots + 56$ . 2
- (b) A geometric sequence of positive terms has first term 2, and the sum of the first three terms is 266. Calculate the common ratio. 3
- (c) An arithmetic sequence,  $A$ , has first term  $a$  and common difference 2, and a geometric sequence,  $B$ , has first term  $a$  and common ratio 2. The first four terms of each sequence have the same sum. Obtain the value of  $a$ . 3
- Obtain the smallest value of  $n$  such that the sum to  $n$  terms for sequence  $B$  is more than **twice** the sum to  $n$  terms for sequence  $A$ . 2

[END OF QUESTION PAPER]