

X056/701

NATIONAL
QUALIFICATIONS
2001

FRIDAY, 25 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. There are five Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2

Section B assesses the optional unit Mathematics 3

Section C assesses the optional unit Statistics 1

Section D assesses the optional unit Numerical Analysis 1

Section E assesses the optional unit Mechanics 1.

Candidates must attempt Section A (Mathematics 1 and 2) **and one** of the following Sections:

Section B (Mathematics 3)

Section C (Statistics 1)

Section D (Numerical Analysis 1)

Section E (Mechanics 1).

3. **Candidates must use a separate answer book for each Section.** Take care to show clearly the optional section chosen. On the front of the answer book, in the top right hand corner, write B, C, D or E.
4. A booklet of Mathematical Formulae and Statistical Tables is supplied for all candidates. It contains Numerical Analysis formulae and Statistical formulae and tables.
5. **Full credit will be given only where the solution contains appropriate working.**

Section A (Mathematics 1 and 2)

Marks

All candidates should attempt this Section.

Answer all the questions.

A1. Use Gaussian elimination to solve the following system of equations

$$\begin{aligned}x + y + z &= 10 \\2x - y + 3z &= 4 \\x + 2z &= 20.\end{aligned}$$

5

A2. Differentiate with respect to x

(a) $f(x) = (2+x)\tan^{-1}\sqrt{x-1}$, $x > 1$,

4

(b) $g(x) = e^{\cot 2x}$, $0 < x < \frac{\pi}{2}$.

2

A3. Find the value of

$$\int_0^{\pi/4} 2x \sin 4x \, dx.$$

5

A4. Prove by induction that, for all integers $n \geq 1$,

$$2 + 5 + 8 + \dots + (3n - 1) = \frac{1}{2}n(3n + 1).$$

5

A5. (a) Obtain partial fractions for

$$\frac{x}{x^2 - 1}, \quad x > 1.$$

2

(b) Use the result of (a) to find

$$\int \frac{x^3}{x^2 - 1} \, dx, \quad x > 1.$$

4

A6. Expand

$$\left(x^2 - \frac{2}{x}\right)^4, \quad x \neq 0$$

and simplify as far as possible.

5

- A7.** A curve has equation $xy + y^2 = 2$.
- (a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y . 3
- (b) Hence find an equation of the tangent to the curve at the point $(1, 1)$. 2
- A8.** A function f is defined by $f(x) = \frac{x^2 + 6x + 12}{x + 2}$, $x \neq -2$.
- (a) Express $f(x)$ in the form $ax + b + \frac{b}{x + 2}$ stating the values of a and b . 2
- (b) Write down an equation for each of the two asymptotes. 2
- (c) Show that $f(x)$ has two stationary points.
Determine the coordinates and the nature of the stationary points. 4
- (d) Sketch the graph of f . 1
- (e) State the range of values of k such that the equation $f(x) = k$ has no solution. 1
- A9.** (a) Given that $-1 = \cos \theta + i \sin \theta$, $-\pi < \theta \leq \pi$, state the value of θ . 1
- (b) Use de Moivre's Theorem to find the non-real solutions, z_1 and z_2 , of the equation $z^3 + 1 = 0$. 5
Hence show that $z_1^2 = -z_2$ and $z_2^2 = -z_1$. 2
- (c) Plot all the solutions of $z^3 + 1 = 0$ on an Argand diagram and state their geometrical significance. 3

[Turn over

- A10.** A chemical plant food loses effectiveness at a rate proportional to the amount present in the soil. The amount M grams of plant food effective after t days satisfies the differential equation

$$\frac{dM}{dt} = kM, \text{ where } k \text{ is a constant.}$$

- (a) Find the general solution for M in terms of t where the initial amount of plant food is M_0 grams. 3
- (b) Find the value of k if, after 30 days, only half the initial amount of plant food is effective. 3
- (c) What percentage of the original amount of plant food is effective after 35 days? 2
- (d) The plant food has to be renewed when its effectiveness falls below 25%. Is the manufacturer of the plant food justified in calling its product "sixty day super food"? 2

[END OF SECTION A]

Candidates should now attempt ONE of the following

Section B (Mathematics 3) on Page five

Section C (Statistics 1) on Page six

Section D (Numerical Analysis 1) on Page eight

Section E (Mechanics 1) on Page ten.

Section B (Mathematics 3)

Marks

**ONLY candidates doing the course Mathematics 1, 2 and 3
should attempt this Section.**

Answer all the questions.

**Answer these questions in a separate answer book, showing clearly the
section chosen.**

- B1.** Use the Euclidean algorithm to find integers x and y such that

$$149x + 139y = 1.$$

4

- B2.** Find the general solution of the following differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = x, \quad x > 0.$$

4

- B3.** Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}.$$

Show that $AB = kI$ for some constant k , where I is the 3×3 identity matrix.
Hence obtain (i) the inverse matrix A^{-1} , and (ii) the matrix A^2B .

4

- B4.** Find the first four terms in the Maclaurin series for $(2 + x) \ln(2 + x)$.

4

- B5.** Find the general solution of the following differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x - 1.$$

5

- B6.** Let L_1 and L_2 be the lines

$$L_1 : \quad x = 8 - 2t, \quad y = -4 + 2t, \quad z = 3 + t$$

$$L_2 : \quad \frac{x}{-2} = \frac{y+2}{-1} = \frac{z-9}{2}.$$

- (a) (i) Show that L_1 and L_2 intersect and find their point of intersection.

4

- (ii) Verify that the acute angle between them is

$$\cos^{-1}\left(\frac{4}{9}\right).$$

2

- (b) (i) Obtain an equation of the plane Π that is perpendicular to L_2 and passes through the point $(1, -4, 2)$.

3

- (ii) Find the coordinates of the point of intersection of the plane Π and the line L_1 .

2

[END OF SECTION B]